Harkins-Brown Correction Factor for Drop Formation

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When a liquid issues at an infinitesimal flow rate from a circular, horizontally-cut, sharp-edged tip of a nozzle into a quiescent, immiscible fluid medium, we will observe a quasi-static growth of a pendant drop perching on the tip. In this case, there should be a balance between the gravitational force acting on the drop to detach it from the nozzle tip and the interfacial tension force acting to keep the drop on the nozzle tip, at an instant preceding the detachment of the drop. Therefore, it follows:

$$V_p \Delta\rho g = \pi D_N \sigma, \tag{1}$$

where

V_p = volume of the pendant drop at the instant
$\Delta\rho$ = magnitude of the density difference between the drop and the medium
g = acceleration due to gravity
D_N = diameter of the nozzle tip
σ = interfacial tension at the drop/medium boundary

The volume of the drop that actually separates from the nozzle tip, V, is smaller than V_p, and is given by

$$V = \psi V_p = \psi \left(\frac{\pi D_N \sigma}{\Delta\rho g} \right), \tag{2}$$

where ψ is a constant smaller than unity and is called the Harkins-Brown correction factor (or Harkins correction factor), after Harkins and Brown (1919) who experimentally found ψ to be independent of the nature of the liquids or the material of the nozzle and to depend only on D_N and V. Harkins and Brown prepared a table that indicates ψ values at regular intervals of $D_N V^{1/3}$ for use in determining the interfacial tension σ by measuring V, the volume of a drop detached from the nozzle. For the next 70 years since Harkins and Brown's work was published, only minor modifications or improvements of the procedure of determining the values of ψ have been made. For example, some later researchers plotted ψ against $D_N(\psi/V)^{1/3}$ instead of $D_N/V^{1/3}$ for use in the prediction of V when σ is known. Note that

$$D_N \left(\frac{\psi}{V} \right)^{1/3} = \frac{D_N}{V_p^{1/3}} = D_N \left(\frac{\Delta\rho g}{\pi D_N \sigma} \right)^{1/3}, \tag{3}$$

hence one can calculate $D_N(\psi/V)^{1/3}$ without knowing V or V_p.

Although the Harkins-Brown correction factor ψ itself seems a celebrated concept, we find that correlations for ψ prepared by the previous researchers have sometimes been used or reproduced erroneously in some later publications, which in turn may have prompted others to miscalculate ψ. In this note, I first cite the erroneous reproductions of some published correlations and then compare them with one another as well as with my newly developed correlation.

Critical Survey of the Literature

Almost a half century after the monumental work of Harkins and Brown (1919), Lando and Oakley (1967) made a regression analysis of Harkins and Brown's data and prepared the following correlation for ψ applicable to a range $0.6 \le D_N/V^{1/3} \le 2.4$:

$$\psi = \frac{1}{2\pi} \left[0.14782 + 0.27896 \left(\frac{D_N}{2V^{1/3}} \right) - 0.166 \left(\frac{D_N}{2V^{1/3}} \right)^2 \right]^{-1}. \tag{4}$$

They stated that 95% confidence limits of the data points about the $(2\pi\psi)^{-1}$ vs. $D_N/2V^{1/3}$ curve given by Eq. 4 were ± 0.0021 in $(2\pi\psi)^{-1}$. Later Heertjes et al. (1971) prepared another curve fitting correlation:

$$\psi = 0.99979 - 1.32045 \left(\frac{D_N}{2V_p^{1/3}} \right) + 1.35743 \left(\frac{D_N}{2V_p^{1/3}} \right)^2, \tag{5}$$

where $D_N/V_p^{1/3}$ can be read as $D_N(\psi/V)^{1/3}$. Heertjes et al. used this correlation for the range $0 \le D_N/V_p^{1/3} \le 0.6$. For use in the complementary range, $0.6 < D_N/V_p^{1/3} < 2.4$, they cited Lando and Oakley's correlation but in a strikingly erroneous form such that

$$\psi = \frac{\pi}{2}\left[0.14782 + 0.27896\left(\frac{D_N}{2V_p^{1/3}}\right) - 0.166\left(\frac{D_N}{2V_p^{1/3}}\right)^2\right]. \qquad (4a)$$

In the first part of his elucidative three-part article on direct-contact heat exchanger design, Obana (1974) reproduced the set of Eqs. 5 and 4a, just as they appeared in the paper of Heertjes et al. (1971), to be combined with Scheele and Meister's kinetic model for drop formation (1968) to calculate the size of drops dispersed with a liquid-injection manifold installed in a heat exchanger.

Clift et al. (1978) correctly rewrote Lando and Oakely's correlation as

$$\psi = \left[0.92878 + 0.87638\left(\frac{D_N}{V^{1/3}}\right) - 0.261\left(\frac{D_N}{V^{1/3}}\right)^2\right]^{-1} \quad \left(0.6 < \frac{D_N}{V^{1/3}} < 2.4\right), \qquad (4b)$$

but they mistakenly replaced V_p with V in the correlation of Heertjes et al. (1971) in rewriting it as

$$\psi = 1.000 - 0.66023\left(\frac{D_N}{V^{1/3}}\right) + 0.33936\left(\frac{D_N}{V^{1/3}}\right)^2 \quad \left(0 \leq \frac{D_N}{V^{1/3}} \leq 0.6\right). \qquad (5a)$$

The set of Eqs. 4b and 5a was then reproduced elsewhere (Hirata, 1982).

Comparison of Correlations for ψ

Scheele and Meister (1968) graphically showed a ψ vs. $D_N(\psi/V)^{1/3}$ relation in the range $D_N(\psi/V)^{1/3} \lesssim 1.55$, but did not express it in a mathematical formula. Horvath et al. (1978) presented the following correlation that they claim approximates Scheele and Meister's graphical plot:

$$\psi = 0.6 + 0.4\exp\left[-2D_N\left(\frac{\psi}{V}\right)^{1/3}\right]. \qquad (6)$$

In their comprehensive review of hydrodynamics of liquid-liquid spray columns, Steiner and Hartland (1983) cited this correlation only.

On the basis of Scheele and Meister's plot, I formulated a new correlation that can be an alternative to Eq. 6 in the range that $D_N(\psi/V)^{1/3} \leq 1.4$, that is

$$\psi = 0.6 + 0.4\left[1 - \frac{D_N}{1.4}\left(\frac{\psi}{V}\right)^{1/3}\right]^{2.2}. \qquad (7)$$

Equations 4–7 and Scheele and Meister's graphical plot are now compared in a $\psi - D_N(\psi/V)^{1/3}$ diagram in Figure 1. Equation 4 is the only correlation whose accuracy was evaluated by the original authors and is most reliable. However, it expresses ψ in terms of $D_N/V^{1/3}$ and hence requires an iterative calculation to determine ψ corresponding to a particular value of $D_N(\psi/V)^{1/3}$. Equations 5–7 can readily be plotted in $\psi - D_N(\psi/V)^{1/3}$ coordinate system. The curve plotted in Scheele and Meister's original $\psi - D_N(\psi/V)^{1/3}$ diagram has been reproduced with the greatest care onto Figure 1.

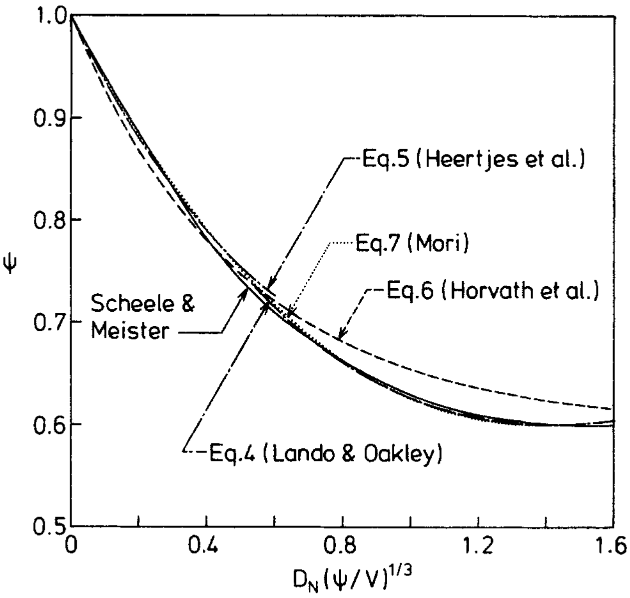

Figure 1. Comparison of correlations for, and Scheele and Meister's graphical presentation of, the Harkins-Brown correction factor.

To identify individual curves running close together, each one is indicated with an arrow pointer of identical line with a relevant index at the end.

Throughout the range of its applicability, Eq. 7 shows an

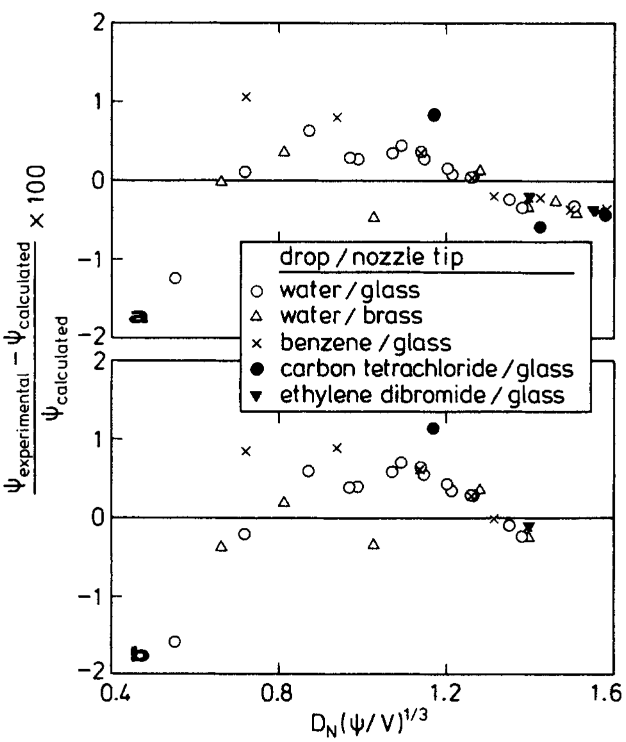

Figure 2. Deviations of Harkins and Brown's experimental data on ψ from ψ values calculated by: a. Eq. 4 and b. Eq. 7.

excellent agreement with Scheele and Meister's curve. Equation 7 also agrees with Eq. 5 very well until $D_N(\psi/V)^{1/3}$ approaches the stated upper limit of the range of applicability of Eq. 5. A close inspection indicates that in the range of $0.54 \leq D_N(\psi/V)^{1/3} \leq 1.4$, Eq. 7 agrees closer with Eq. 4 than with Scheele and Meister's curve. Equation 6 yields a curve appreciably deviating from Scheele and Meister's curve and also those representing the other correlations.

Deviations of Harkins and Brown's original data, which fall in the range $D_N(\psi/V)^{1/3} \gtrsim 0.55$, from ψ values calculated by Eq. 4 and Eq. 7, are shown in Figure 2. It is found that in the range $D_N(\psi/V)^{1/3} \leq 1.4$, Eq. 7 represents Harkins and Brown's data almost as closely as Eq. 4. This fact suggests that Eq. 7 is the best compromise between accuracy and ease of use for predicting the Hawkins-Brown correction factor in the abovementioned range of $D_N(\psi/V)^{1/3}$.

Acknowledgment

I would like to thank N. Ehara, a student at the Department of Mechanical Engineering, Keio University, for his help in figure preparation.

Notation

D_N = nozzle diameter
g = acceleration due to gravity
V = volume of detached drop
V_p = volume of pendant drop at instant of force balance given by Eq. 1

Greek letters

$\Delta\rho$ = density difference between drop and surrounding medium
σ = drop/medium interfacial tension
ψ = Harkins-Brown correction factor

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Manuscript received Mar. 6, 1990, and revision received May 18, 1990.